Structures Research Section

TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 702

DETERMINATION OF INHERENT STRESSES BY MEASURING DEFORMATIONS OF DRILLED HOLES

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Archiv für das Eisenhüttenwesen
Vol. VI, No. 7, 1932-33

Washington
March, 1933

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DETERMINATION OF INHERENT STRESSES BY MEASURING DEFORMATIONS OF DRILLED HOLES*

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Various methods have been proposed for determining the inherent stresses in structural components by disturbing their stress equilibrium through some mechanical device and measuring the resulting deformations. This principle is the basis of the stress methods of E. Heyn and O. Bauer,** the casting of stress grids for determining the tendency of various cast irons to develop stresses*** and the drilling methods of G. Sachs.**** The working methods have the disadvantages, however, that they can be successfully used only with specially shaped pieces (e.g., those with round or rectangular cross sections), that every form of test piece requires another kind of injury and hence of calculation, and that the tested parts are rendered useless. In part, moreover, only mean stresses can be determined which may differ greatly from the maximum stresses.

The new test method, which seeks to eliminate these disadvantages, is likewise based on a disturbance of the equilibrium of forces, and indeed by drilling a hole which, however, is so small that the part can be used again. This method serves, among other things, for determining the inherent stresses in castings, welded parts, rolled structural shapes and finished structures.

In order to explain the fundamental principle, it is at first assumed that the part to be tested, which is

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*"Ermittlung von Eigenspannungen durch Messung von Bohrloch-Verformungen." Archiv für das Eisenhüttenwesen, Vol. VI, No. 7, 1932-33, pp. 277-281. Communication from the Aachen Aerodynamic Institute.

**Stahl und Eisen: Vol. 31, 1911, pp. 760-765.

***R. v. Steiger: Dissertation (Zürich, Gebr. Leemann, 1913.) Compare Stahl und Eisen, Vol. 33, 1913, pp. 1442-1443.

****Z. Metallkunde, Vol. 19, 1927, pp. 352-357.

very wide, is subjected to a known stress, which is uniform throughout the thickness and has a single permanent directional axis. A tensometer is then placed on this test piece in the direction of the stress. If a hole is now drilled between its foot points a and b (fig. 1a), this hole will become an ellipse under tensile stresses and the distance between the points a and b will be increased. If the relation between this distance and the stress is determined by calculation or by a calibration test, then the stress in the test piece in the direction a b can be calculated from the increase in the distance between a and b.

Figure 2 shows, by way of example, how the distance between the points a and b is affected by the penetration of the drill into the material. The form of the drill, which chiefly affects the course of the curve, was determined experimentally on the basis of the smoothest possible operation. When the tip of the small drill penetrates the test piece, the change in the distance must be extremely small, since the resulting conical hole is small and far from the points. It is still small when the cylindrical part of the first drill penetrates. Yet when the main drill begins to cut, the change in the distance suddenly increases, as shown by the break in the curve, but then increases more slowly until the exit of the main drill. In a thick piece, the increase in the distance constantly diminishes to zero, i.e., it approaches a limiting value, since all the stresses which are liberated through the removal of the material by the drill, have no appreciable effect on the deformation of the surface. From this fact it follows that it is not necessary to drill clear through thick pieces to determine the stresses. The tests showed that the depth of the holes needed to be only 1.5 to 2 times their diameter.

In testing a piece which is known to have a uniform condition of stress according to the depth, it is, of course, not absolutely necessary to plot the whole distance increase against the depth of the hole. Its determination at the end of the drilling suffices, although, to be on the safe side, it is always advisable to plot the whole curve.

For the basic principle of the test method, it does not matter whether the increase in the distance between the points a and b according to Figure 1a or between the points a' and e according to Figure 1b is measured. In the latter case, if the point d were infinitely distant from d, the distance increase would be just half that between a and b. In practice, the point e is not very far from a', since the test values at a finite distance rapidly approach those at an infinite distance. In the experimental arrangement this distance was 15 cm (5.91 in.).

If the test piece is subjected not to a monaxial, but to a biaxial state of stress, then one measurement is not enough, but the deformation of the hole must be measured in three different directions, in order to determine the magnitude and direction of the maximum and minimum main stresses. If only one measuring instrument is available and it is known that the stress remains constant over a large area, three holes can be drilled in this area and each be measured at a different angle.

If the stress varies within the depth of the hole, as, e.g., in bending or surface stresses, the course of the curve gives qualitative indications regarding these changes. Further details will be given elsewhere.

The diameter of the drill used in the test apparatus was 12 mm (0.472 in.). It might, of course, be larger or smaller. The upper limit resides in the requirement of the least possible weakening, while the lower limit is determined by the accuracy of the measuring apparatus. For laboratory apparatus, the diameter could be reduced to 6 mm (0.236 in.). This is the limit, however, since otherwise the transmission ratio of the measuring instrument would be too large.

After the test, a rivet or plug can be inserted in the hole, according to whether the hole goes clear through or only part way. A tightly fitting rivet will reduce notch stresses from subsequent loading, since the effect of the rivet head and its pressure on the walls of the hole will partially prevent the deformations which would occur with an open hole. The supporting strength of the structure or the utility of the part will seldom be affected by the slight weakening produced by the test. If, e.g., the test is made on a girder, a 12 mm hole would have little effect in comparison with the many larger holes required for assembling.

The calibration of the measuring device, i.e., the determination of the relation between the final elongation of the test distance and the stress, is made, once for all, for the principal material. So long as the stresses are less than 40 per cent of the proportionality limit, they are proportional to the final elongations. (Fig. 3.) The calibration can then be made also by calculation. Above this point it must be made by experimentation. The fact that the stresses, which can be calculated, are so low, is due to the high notch stresses at the edge of the hole.

The calibration of the measuring device by calculation is based on a report by Hirsch,* who calculated the elongation of the holes in members of infinite width in terms of the tensile stress, and on a report by Willheim and Leon,** who extended this method approximately to members of finite width. The relations established by Kirsch, Willheim and Leon, which, as developed for the monaxial state of stress, could be directly extended to the biaxial state, can be expressed as a calibration curve, if the uniform elongation corresponding to the stresses at the time is subtracted from their values of the distance increase. This subtraction is necessary, because the measuring device is installed when the piece is already stressed by the uniform tension and therefore the points a and b in Figure 1a have already undergone an increase in distance from the beginning of the test corresponding to this tension.

The experimental calibration can be made by mounting a broad flat plate with a 12 mm (0.472 in.) hole in the tensile machine. Then a tensometer, preferably the one used in the hole tests is so mounted on the plate that its measuring points rest on the points a and b according to Figure 1a. The plate is then stressed and the resulting increase in the distance between a and b is measured. Of course the resulting relation does not represent the final calibration curve, but there must first be subtracted from the distance increase the amount which the measurement would yield if the hole did not exist. The resulting calibration curve is valid only for plates of the width used in the test. After calibration curves have been plotted for plates of different widths, the values for plates of infinite width can be extrapolated.

The calibration test can also be made like the subsequent investigation, excepting that the distance in-

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*Z.V.D.I., Vol. 42, 1898, pp. 797-807.
**Z. Mathematik und Physik, Vol. 64, 1916, p. 233.

crease is measured in terms of the known stress. A flat plate is therefore subjected to a known static load, the measuring instrument is installed, and the hole is drilled. The increase in distance between a and b is thus determined according to Figure 1a in terms of the depth of the hole and therefore the total increase corresponding to the stress. If this test is repeated for a series of different stresses, a series of curves is obtained from which the final calibration curves, i.e., the distance changes in terms of the stresses, can be determined. If, in the material under investigation, the stress-strain line for tension differs from that for compression, the test must be made for each. The width of the plate will be taken into consideration in the same way as in the calibration test already described.

While both calibration tests can be used under 40 per cent of the proportionality limit, a special calibration curve must be plotted according to the second test for every material in accurate investigations above this limit, since, due to exceeding the proportionality limit, the resulting deformations will no longer be entirely independent of the manner of making the hole. In general, however, it will suffice in the experimental calibration to use the first calibration test and to plot the calibration curves for whole groups of the materials, e.g., in the case of steel, for steel with small, medium and great elongations.

Figure 4 is a picture of the instrument which records the increase in the distance between a and b in Figure 1. m and n are the two points of the extensometer which works like the Martens reflecting instrument. Two auxiliary devices, a clamp k and a centering bush r, keep the test distance exact and both foot points at the same distance from the hole. Both devices are loosened after the installation of the stirrup-cutter assembly, in order that the cutter may turn freely and that the stirrup will not be affected by the motion of the drill s. The sleeve v serves to center the bush r and to carry away the drillings.

Since the installation of the telescope required for this test is often difficult, an indicator has been developed, in which the index is actuated by the variation in the distance. This indicator does not span the hole like the reflecting instrument, but it rests on the points a', b', c' of the test piece. (Fig. 1b.) The test point is at the edge of the hole, and both fixed supports are about 15 cm (5.9 in.) from the hole. Figure 5 shows the whole test apparatus with this indicator q, which has a transmission ratio of about 1:3200. The transmitting lever is balanced in every position of the pointer. The measuring instrument is clamped to the test place by the arm u on the drilling machine z. The drillings are carried away by a sleeve t. The 0.3 hp drilling machine is driven by a flexible shaft and runs very smoothly. The drilling pressure is kept nearly uniform by the interposition of a spring. The depth drilled is measured by a Zeiss clock v. The drilling machine is clamped to the test plate by the test frame f.

According to the method described, two H beams NP 20 with a length of 6 cm (about 20 ft.), one iron I beam NP 20 of the same length and an iron channel NP 26 (also of the same length) were tested for their inherent stresses. In the H beam the inherent stresses were determined over the whole length of the web (fig. 6) and in both flanges at the points of maximum web stresses. (Fig. 7.) For every test point the full elongation curve of the test length was plotted against the depth of the hole. All the places were drilled clear through with the exception of the middle of the flange. The curves, almost without exception, resemble Figure 2, from which it may be concluded that the stresses hardly vary throughout the thickness. For this reason and the consideration that chiefly a monaxial stress must exist in the girders, the calibration curve of this stress condition was taken as the basis for the stress determination. There were extremely high tensile stresses in the middle of the webs of both beams in the direction of the webs, amounting in the middle of the girders to about 20 kg/mm^2 (28,447 lb./sq.in.). At this stress, the index was deflected about 36 mm (1.42 in.). The maximum stress was in the central part of each beam. At both ends, as was to be expected, the stresses dropped to zero. Of course the measurements on the two beams did not agree, either in their course or in their maximum stresses, in which they differed by about 10 per cent. The dissimilarity in the stress curves at the ends of beam II is ascribable to the fact that the left end remained as rolled and the right one was cut off. The flanges were stressed in compression, the stresses being small in the middle and increasing toward the ends. The analysis of the test results showed that the sum of the moments in a cross section was almost zero.

The stresses were considerably smaller in the I beams than in the H beams. (Fig. 8.) The stresses were determined only in the middle of the web, the tests showing stresses of about 2 kg/mm² (2,845 lb./sq.in.). In the iron channel the stresses were likewise determined for the middle of the web. In general the stresses were relatively small. (Fig. 9.) In the middle there occurred a sudden increase, which was probably due to overstressing in mounting. General conclusions could not be made directly from the test results. For example, the results of the stresses in the H beams were contrary to the theoretical calculations by Ros, who, however, considered only the cooling relations.*

For checking the test results, the H beam II was sawed in the middle of the web at the right end for a distance of about 50 cm (about 20 inches). If there were tensile stresses in the web, as indicated by the test, the halves of the beam after sawing would approach each other. According to Figure 10 the ends separated a little at first, but drew strongly together again after being sawed 5 cm (about 2 inches). After sawing 48 cm (about 19 inches), the separation of the ends had diminished about 3 mm (0.12 in.). The second test consisted in sawing out, at a distance of 2.6 m (8.53 ft.) from the right-hand end of the H beam I, a strip 32 by 4 cm (about 12.6 by 1.57 in.), as shown in Figure 11. If there were tensile stresses of about 20 kg/mm² (28,447 lb./sq.in.) at this place in the web, the sawed-out strip should have contracted about 0.29 mm (0.0114 in.) according to the formula

$$\text{contraction} = \frac{\text{stress}}{\text{Young's modulus}} \times \text{test length.}$$

The test showed a contraction of 0.27 mm (0.0107 in.). The results of the proposed method are thus confirmed.

Lastly, attention is called to the following point. Since the beams have inherent stresses, care must be taken to eliminate these stresses in testing beams supporting the weight of a structure for the determination of the developed stress σ_g. This can be accomplished either by drilling the beam section at various points and calculating

*Bericht 52, Eidgen. Mat.-Prüf.-Anst. Techn. Hochsch., Zürich, 1930.

σ_g from the formula

$$\sigma_g = \frac{1}{F} \int \sigma(\text{measured})\, df$$

($F \doteq$ area of beam section), or by drilling the beam where the inherent stresses are presumably zero, as, e.g., in the middle of the flange of an H beam.

SUMMARY

For determining the inherent stresses in monaxial and biaxial conditions in test pieces which show equal stresses in a layer about 2 cm (0.79 in.) thick, it is proposed to drill a small hole in the test piece and to measure the resulting deformations in the immediate vicinity of the hole. For known monaxial stress conditions, it is only necessary to measure the change in the distance between two diametrically opposite points. The stresses in the test piece can be determined from these changes in distance. The apparatus accordingly consists of a small drilling machine, driving, by means of a flexible shaft, a special drill whose maximum diameter is limited by the permissible weakening of the test piece and whose minimum diameter is limited by the accuracy of measuring, and an extensometer for which a special indicator is used along with the Martens reflecting instrument. The tested parts can generally be used further, after filling the holes with rivets or plugs.

Two H beams were tested by this method, the tensile stresses in the web and the compressive stresses in the flanges being determined. The results were confirmed by slotting the beam and by cutting a strip out of the web. Nearly uniform compressive stresses were found in an iron I beam and alternating compressive and tensile stresses in an iron channel. Of course generalizations cannot be made from these results.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

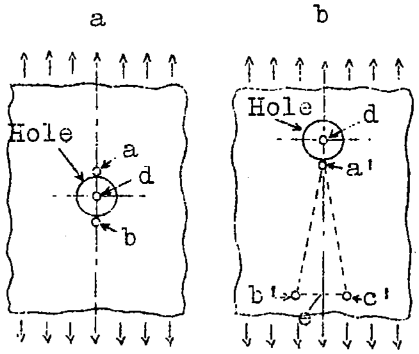

Figure 1.- Arrangement of reflecting and indicating instruments for measuring hole deformations.

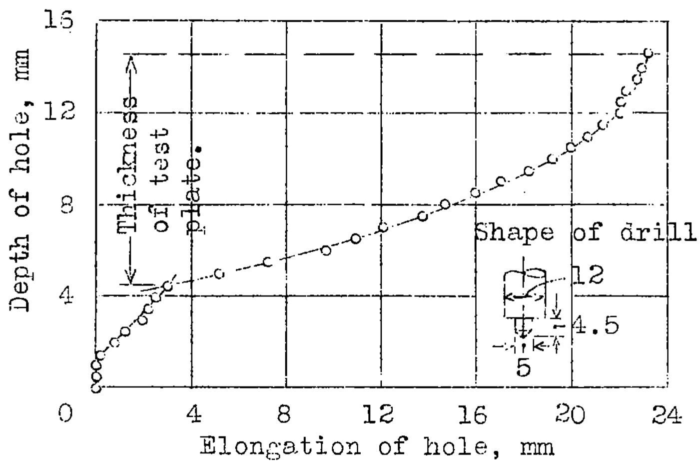

Figure 2.- Depth against elongation of hole (mm).

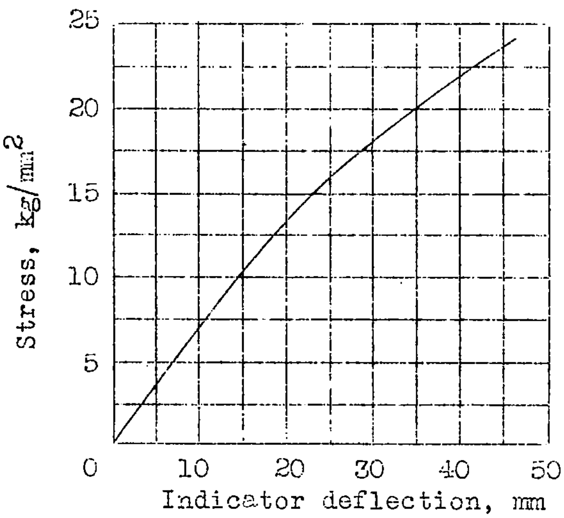

Figure 3.- Calibration curve for a soft steel.

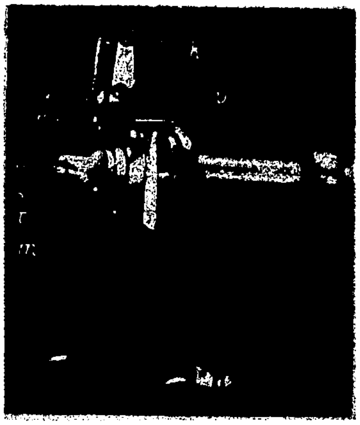

Figure 4.- Test apparatus with reflecting extensometer.

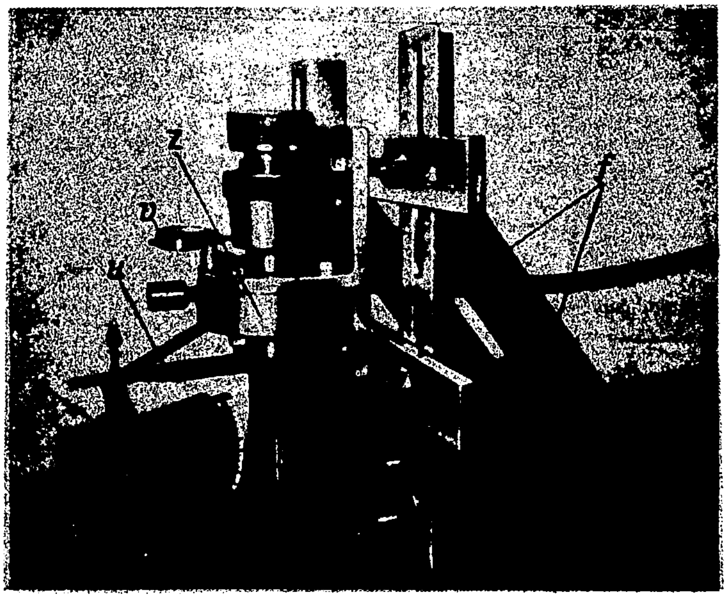

Figure 5.- Apparatus for using indicator.

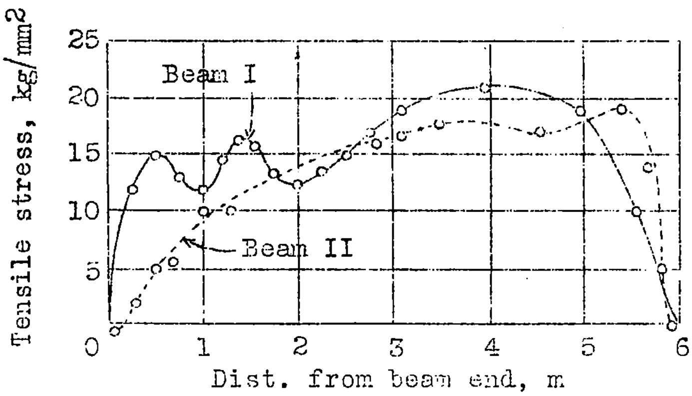

Figure 6.- Distribution of inherent stresses over length of H beam NP 20, measured in middle of web.

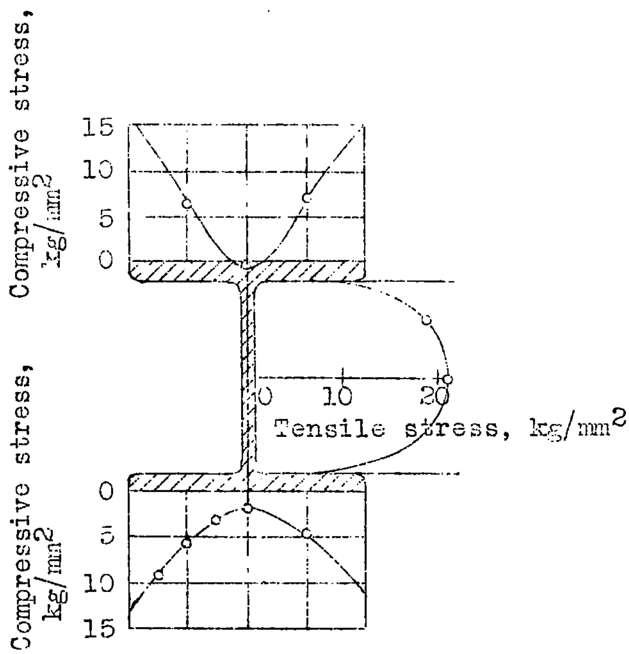

Figure 7.- Distribution of inherent stresses over cross section of an H beam NP 20.

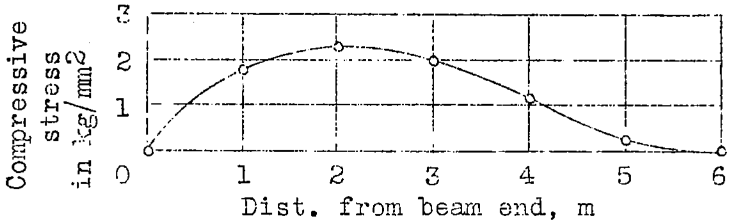

Figure 8.- Distribution of inherent stresses over length of I beam NP 20, measured in middle of web.

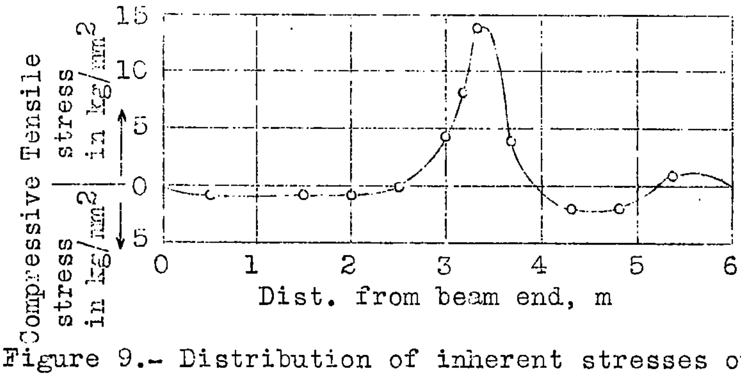

Figure 9.- Distribution of inherent stresses over length of channel NP 26, measured in middle of web.

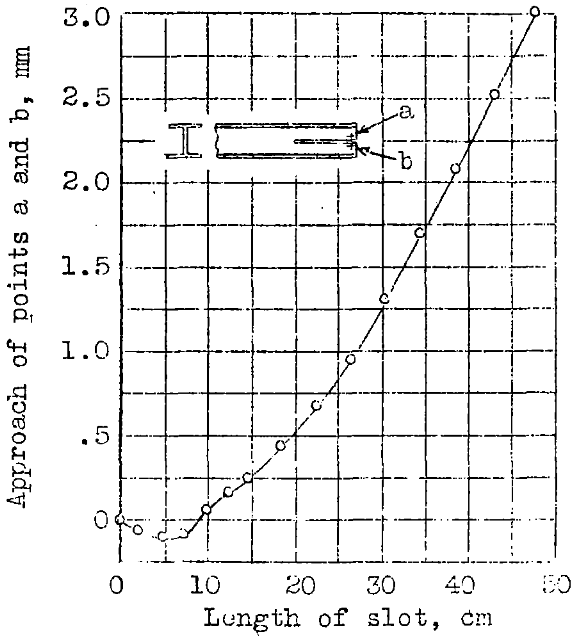

Figure 10.- Approach of points a and b due to slot in web middle at right end of H beam II.

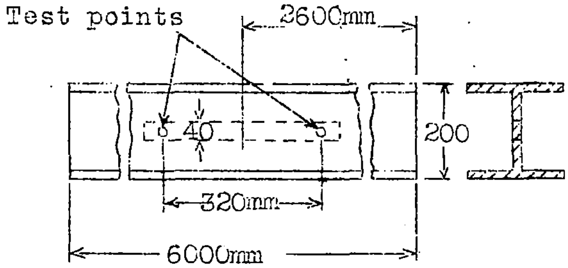

Figure 11.- Location and shape of test strip in H beam I.